A Simple Model for Understanding Cloud Diffusion on a Brown Dwarf

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Abstract. Brown dwarfs in the L-T spectral class transition commonly experience photometric variability due to the active formation/dissipation of clouds that rotate in and out of our view. Measurements of these photometric oscillations, such as their frequency and amplitude, may help constrain the physical parameters of observed brown dwarfs through their associations with aspects such as rotational period and surface temperature. However, measurements of these oscillations and their significance are obscured by the inclination angle of observed brown dwarfs relative to us. By creating a simplistic model of 2D cloud formation on the surface of a toy model brown dwarf, this paper aims to further explore the relationship between oscillation amplitude and inclination angle for cloudy brown dwarfs and finds agreement with the correlation found observationally between the two factors in Vos et al., 2017.

INTRODUCTION

Brown dwarfs are substellar objects in a mass range 0.012-0.075 $M_\odot$ and surface temperature range <2400 K (classified by the L, T, Y system; L being the hottest, Y being the coolest), with no theoretical lower limit for their surface temperature. However, the faintness of colder brown dwarfs, such as Y dwarfs typically below 400 K, makes their observation difficult, with the coldest known brown dwarf WISE J0855 being at 250 K [1]. Brown dwarfs form from the collapse of interstellar material, similar to a star, but do not gather enough mass to perform stable H to He fusion in their core. They instead undergo an insignificant amount of deuterium fusion due to their lower pressure/temperature threshold. Due to their severe lack of energy production, they slowly cool as they radiate their internal energy away [2, 3].

Brown dwarfs below 2200 K are often cool enough that atoms normally ionized in stars can form complex molecules. These molecules may even condense into a liquid/solid form on seed particles, creating clouds that stretch across the brown dwarf's atmosphere. Brown dwarfs transitioning between the L-T phase observationally are in a particularly active state of cloud formation and dissipation. These brown dwarfs are cool enough to support cloud condensation and have the more massive condensate grains, making the clouds eventually sink below the photosphere. The consequence is that "patchy" clouds constantly form, break up, and sink across the surface of L-T brown dwarfs [4, 5].

As a result of these clouds, photometrically, brown dwarfs appear to have an oscillating flux over time as a result of features rotating in/out of view and forming/dissipating during an observation [6]. This is especially true for L-T brown dwarfs due to their continual formation and dissipation of patchy clouds. Measuring the frequency and amplitude of these oscillations could provide valuable constraints on parameters for an observed brown dwarf, such as finding a potential relationship between the amplitude of oscillations and the spectral type of a brown dwarf.

A significant hurdle in this prospect, and one this paper aims to help resolve, is that the inclination angle of a brown dwarf relative to us drastically alters our photometric perception of surface features. Inclination potentially affects the flux received (or blocked) from clouds by changing their effective area, introducing limb darkening effects or changing how long they will stay in our field of view during one rotation [6]. By adopting a crude 2D toy model of a brown dwarf's surface cloud structure that includes limb darkening, convection, diffusion, and rotational effects, we attempt to find a tangible relationship between inclination angle and flux oscillation amplitude to explore this issue further. Additionally, we compare our results with the amplitude/inclination relation predicted in a previous paper by Vos et al., 2017.

METHODS

We wish to describe the time evolution of clouds on a brown dwarf as a function of relevant physical phenomena: diffusion, convection, rotation, and the Coriolis effect. Though these forces have been included in brown dwarf atmosphere models in the past [7, 8], we are not aware of them having been isolated in the form we present. The proposals we give here are intentionally simplified; our goal is to encapsulate the fundamental behavior of how these should act when superimposed on a brown dwarf without getting caught in an overcomplicated analysis.

The diffusion of any material across a surface is a well-studied phenomena, leaving us with no need to reinvent the proverbial wheel [9].

$$\frac{du}{dt} = D\nabla^2 u = \frac{D}{R^2 \sin(\theta)} \left[\partial_\theta(\sin(\theta)\partial_\theta u) + \partial_\varphi^2 u\right] = F_1(t,\theta,\varphi) \tag{1}$$

The time evolution of a gas in a space is governed by the Laplacian of that gas's concentration. In our case, u will describe the concentration of a "cloud seed" particulate (dust grains), and we will study the diffusion on the surface of the brown dwarf. R is the radius of the brown dwarf, θ and φ are the latitudinal and longitudinal coordinates, respectively. We simulated with a normalized radius of $R = 1$.

Brown dwarfs are known to have cool, convective atmospheres composed of thin adiabatic layers [7]. There has been success rigorously modeling the subsequent dust formation and mixing in the atmosphere, with the field of *mixing length theory* [8]. Such hydrodynamic models capture the essentials of radiative transfer and energy transport in three dimensions.

We wish to encapsulate the qualitative behavior of dust mixing due to convection with a simple model that neglects the finer complexities of convection. In summary, this behavior is that an object with a strong convective outer layer tends to form granules across its surface; Schwarzschild defined these as the visible tops of rising convective elements [10]. We claim that dust particles rise within a granule with some constant frequency and that these granules are evenly distributed across the surface of the brown dwarf. We acknowledge the crude nature of such a claim while hoping that it does justice to convection to some approximation. Hence, due to convection,

$$\frac{du}{dt} = A\sin(\omega t + \theta + \varphi) = F_2(t,\theta,\varphi). \tag{2}$$

A and ω are the convection amplitude and timescales, respectively. This paper does not explore parameter dependencies; that is, we assume each force at play operates with weight on the same order of magnitude (i.e., $D = A = 1$, $\omega = 1$). While this choice is able to reproduce essential behavior, a future direction could be to search for bifurcations as these are varied.

Rotation will result in the physical movement of dust particulate across longitude values, and the Coriolis effect should deflect longitudinal motion towards the equator. Rotation will have the strongest impact near the equator, where particulate must travel at a faster speed to keep up. In contrast, the Coriolis effect is zero at the equator.

To put this into mathematical language, consider the numerical representation of this phenomena. At any time step, t, rotation must move particulate from one longitude value into an adjacent longitudinal value. Thus, the change in particulate concentration at a given longitude is proportional to the adjacent longitude's particulate concentration and is inversely related to that longitude's own concentration

$$u[\varphi]_{t+\Delta t} = u[\varphi]_t + \gamma(\theta)(u[\varphi - \Delta\varphi]_t - u[\varphi]_t) \qquad \gamma(\theta) = \frac{1}{30\sqrt{2\pi}} \exp\left[((\theta - 90)/30)^2\right] \tag{3}$$

where we took γ as a normal distribution maximized at the equator. We can complete this argument by converting to a continuous time domain and incorporating a similar argument for the form of the Coriolis effect [11]

$$\frac{du}{dt} = -\gamma(\theta)u_\varphi + [H(\theta - 90^\circ)\sin(\theta - 90^\circ) - H(90^\circ - \theta)\sin(90^\circ - \theta)]\,u_\theta = F_3(t,\theta,\varphi) \tag{4}$$

where H is the Heaviside function (which is incorporated to account for the difference in the Coriolis effect between the two hemispheres).

The resultant equation describing the time evolution of dust particulate is given by the superposition of Eqs. (1)-(4), that is, $\frac{du}{dt} = \sum_i F_i(t,\theta,\varphi)$, where each F_i is a distinct atmospheric force. The solution $u(t,\theta,\varphi)$ outputs information regarding where particulate and clouds will accumulate. We simulate with a forward Euler numerical integration scheme [12]. Specifically, we solve an initial value problem starting with a uniform dispersion ($u(0,\theta,\varphi) = u(0)$).

CONCLUSIONS

The results of this approach are given in Fig. 1(a), with a video link to illustrate time dependence. Convection applies a visible forcing at each latitude, longitude coordinate. Rotation and the Coriolis force induce a global behavior of particulate movement around the sphere, with a buildup near the equator.

We wish to use our simulated predictions regarding the particulate and subsequent cloud concentrations in order to make claims regarding the relationship between viewing angle and light curve amplitude of a brown dwarf. To do this will require a few points of machinery to be explored. From our model, we can extract light curves resulting from any given set of latitude values. These are taken by assuming the cloud concentration at a point is inversely related to escaped light from the brown dwarf at that coordinate. Thus the simulation can equivalently be viewed as telling us the emissivity as a function of latitude, longitude, and time.

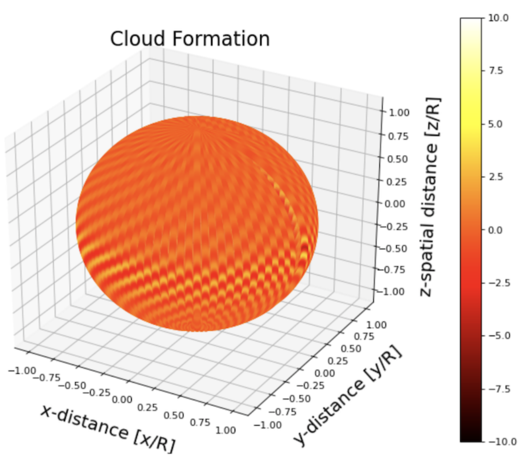
What an astronomer would see when viewing any stellar or large substellar object is a fraction of the light over the aligned hemisphere. However, integrating over a full 180° surface uniformly would be a misguided excursion neglecting the impact of limb darkening. We can summarize this effect in the following way: if the dot product of a normal vector to the surface at a given coordinate with a unit vector in the direction of the astronomer's line of sight is small, then a photon coming from that coordinate will have to travel through more atmosphere than an equivalent photon coming from a point where the dot product is 1 [13]. Hence, in extracting results, we take care to down weight the influence of the outer limbs of a surface of interest.

Figure 1(b) illustrates how the relationship our model predicts between amplitude variability and inclination compares with preexisting data. Before moving further, we will note our choice of normalization in using a z-score norm. We assume the variability in the light curve amplitudes to be loosely distributed as a Gaussian, and we report the number of standard deviations away from the mean the light curve amplitude is when viewed at each fixed inclination angle. This is a toy model that cannot reproduce light curves exactly, but we can normalize it to view a relative comparison with available data.

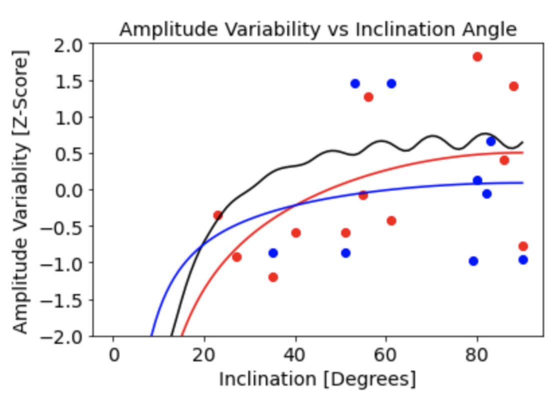
The presented curve fits are in the form suggested by Vos, et. al., 2017:

$$A = \alpha \sin(\theta) - \frac{\kappa}{\sin(\theta)}. \tag{5}$$

The key prediction of our model that comes with a reasonably strong observational justification is that observed amplitude variability is limited for brown dwarfs with a small angle of inclination to the observer. Notably, we are of the understanding that there is no observational evidence of a brown dwarf with both a small ($< 20^{\circ}$) inclination angle and a non-negligible amplitude variability [6].



(a)



(b)

FIGURE 1. (a) A fixed time image of the simulation. The color corresponds to the number of standard deviations a given spatial coordinate is away from the mean particulate concentration. View the full simulation online at https://www.youtube.com/watch?v=GrIyYdXeHaU. (b) Amplitude variability for a brown dwarf plotted against inclination angle. The black curve shows the results of our simulation. The red data (and subsequent curve fit) refers to *Spitzer* 36 μm detections. The blue data (and subsequent curve fit) correspond to *J*-band detections.

Our model provides a baseline explanation for these observations. The atmospheric forces at play in a brown dwarf result in cloud oscillations being greatest near the equator; hence, the resultant light curve from the equator exhibits the greatest variability. Limb darkening minimizes this effect if the brown dwarf is viewed at a small angle of inclination. In such a case, the light curve would exhibit minimal variability as clouds sit still at the poles.

Another explanation for the effect could potentially be provided from the Doppler effect. If the brown dwarf is viewed equator on (high inclination angle), then we would expect wavelength deviation from light across the brown dwarf, resulting in variability when viewing the brown dwarf in a specific band. Such a theory is beyond the scope of this model's capacity but has been explored by other researchers [14].

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