

Test Particle Calculation of Plasma Equilibria

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Abstract. We present a numerical technique for self-consistently calculating plasma equilibria with prescribed sources and sinks on the boundaries, i.e. a scattering system. The method is applied to the earth's magnetotail. The method follows individual particles through a prescribed magnetic field, while calculating the density, current and pressure that the particle contributes on a uniformly spaced grid. The individual particles are weighted to model a given source distribution and the total equilibrium properties, including the resulting magnetic field, are evaluated. The calculated and prescribed magnetic fields are then compared. If the fields differ significantly, the two fields are mixed and the process repeated. Convergence to the self-consistent field typically takes between 100 and 150 iterations.

INTRODUCTION

Determining the self-consistent properties of a plasma equilibrium is typically a very challenging problem, but is essential for determining the stability and wave properties in the plasma. This is particularly true for cases where the scale length of the variations in the plasma are on the order of the ion gyroradius or smaller. One common method is to choose a distribution function and self-consistently solve the Vlasov-Maxwell equations using moments of the distribution. This technique has produced many useful results,¹⁻⁵ however it misses the effects of nonlinear/chaotic particle dynamics on the distribution function and resulting equilibrium and stability. Test particle simulations provide an alternative, in which the fields are assumed to be known and a distribution of particles is launched from a source region and pushed through the given fields.⁶⁻⁹ Once all the particles in a source distribution have moved through the system, the total average density (n), current (\mathbf{j}), pressure tensor (\mathbf{Q}), are used to update the fields. The process is repeated until the input and calculated fields are in agreement. Test particle codes are well suited for determining equilibrium solutions with good spatial resolution using a relatively small number of particles, but cannot yield any information about the time evolution and stability of the calculated equilibria. The resulting equilibria may be used as initial conditions for Particle-in-Cell simulations that can evaluate the wave and stability properties of the system.

TEST PARTICLE SIMULATION

A fundamental first step in a test particle code is to compute the contributions of a single particle. This process is complicated by the facts that the particle motion is in general chaotic and that it is not known a priori how long a particle will remain in the calculation region. At any given time, however, the distribution function of a single particle is formally given by

$$f_{sp} = \frac{1}{L^3 \Omega_0^3} \delta\left(\frac{r-r(t)}{L}\right) \delta\left(\frac{v-v(t)}{\Omega_0 L}\right) \quad (1)$$

where L is the characteristic scale length and Ω_0 is the characteristic frequency. The spatial (velocity) delta function is three dimensional so that $\delta\left[\frac{r-r(t)}{L}\right] = \delta\left[\frac{x-x(t)}{L}\right] \delta\left[\frac{y-y(t)}{L}\right] \delta\left[\frac{z-z(t)}{L}\right]$ and similar for the velocity space

delta function. For our work on the current sheet, L and Ω_0 are taken to be the initial scale length of the assumed field and the cyclotron frequency in the asymptotic magnetic field. The first three moments of the distribution (density(n), current(\mathbf{j}), and pressure (\mathbf{Q})) are calculated as

$$n(\hat{r}, t) = \int f_{sp}(r, v, t) dv = \frac{1}{L^3} \delta(\hat{r} - \hat{r}(t)) \quad (2)$$

$$\mathbf{j}(\hat{r}, t) = q \int v f_{sp}(r, v, t) dv = \frac{q\Omega_0}{L^2} \hat{v} \delta(\hat{r} - \hat{r}(t)) \quad (3)$$

$$\mathbf{Q}(\hat{r}, t) = m \int v v f_{sp}(r, v, t) dv = \frac{m\Omega_0^2}{L} \hat{v} \hat{v} \delta(\hat{r} - \hat{r}(t)) . \quad (4)$$

Here q (m) is the charge (mass) of the particle and we have defined the normalized variables $\hat{r} = r/L$ and $\hat{v} = v/\Omega_0 L$. The contribution of a single particle to the equilibrium quantities is obtained by averaging instantaneous values along the trajectory for the time, T , that the particle is in the system, i.e.

$$\overline{W}(\hat{r}) = \frac{1}{T} \int_0^T W(\hat{r}, t) dt, \quad (5)$$

where W is any velocity moment (n , \mathbf{j} , \mathbf{Q}) and in general, T will be different for each particle. Assuming that we calculate the particle position and velocity at equally space time intervals, Δt so that $T = N\Delta t$, we may approximate the integral as a finite sum. Thus

$$\overline{n}(\hat{r}) = \frac{1}{L^3} \left[\frac{1}{N} \sum_{n=0}^N \delta_{\hat{r}, \hat{r}_n} \right] \quad ; \quad \overline{\mathbf{j}}(\hat{r}) = \frac{q\Omega_0}{L^2} \left[\frac{1}{N} \sum_{n=0}^N \hat{v}_n \delta_{\hat{r}, \hat{r}_n} \right] \quad ; \quad \overline{\mathbf{Q}}(\hat{r}) = \frac{m\Omega_0^2}{L} \left[\frac{1}{N} \sum_{n=0}^N \hat{v}_n \hat{v}_n \delta_{\hat{r}, \hat{r}_n} \right] \quad (6)$$

where \hat{r}_n (\hat{v}_n) is the normalized position (velocity) at the n^{th} time step and $\delta_{\hat{r}, \hat{r}_n}$ is the Kronecker delta function. Since the moments will be interpolated onto a grid, it is important that the time step be chosen sufficiently small so that a particle does not cross a complete grid cell in a single step. The sums in the square brackets are easily calculated by linearly interpolating the values at each step onto the grid points that bound the cell the particle is in during a particular step. An extra (guard) cell adjacent to the calculation region must be included to ensure the proper contributions of the moments to the edge grid points. In principle this process can be applied in 1, 2, or 3 dimensions.

As a particular example of the test particle method, we consider the magnetotail current sheet where we use the GSE coordinate system with its origin in the center of the earth, the x-direction is in the direction of the sun, y is in the dawn to dusk direction and z is normal to the ecliptic. In this system, the magnetic field is given by $B = B_0 f(z/L) \hat{x} + B_z \hat{z}$ where $f(z/L)$ is a smoothly varying function that asymptotes to ± 1 as $z \rightarrow \pm \infty$. For the case $f(z/L) = \tanh(z/L)$, this is the well-known modified Harris model. The field is taken to only vary in the z-direction, since the scale lengths in x and y are much longer than L.

To calculate the equilibrium profiles, we choose the ion source distribution function in the asymptotic region to be a drifting Maxwellian, i.e

$$f(v) = C e^{-\frac{mv^2}{2T} - \frac{m(v_{\parallel} + v_D)^2}{2T}} \quad (7)$$

where C is a constant, T is the ion temperature and $v_D = \sqrt{\frac{2\varepsilon T}{m}} = \sqrt{\frac{2E_{drift}}{m}}$ is the drift velocity along the field line given in terms of the fraction, ε , of the thermal energy and $E_{drift} = \varepsilon T$. Once a particle has escaped from the system, the single particle moments are weighted such that the density at the top grid point due to that incoming particle is unity to ensure that each particle has equal weight in the phase space. To guarantee that a particle contributes properly to the top grid cell, it is launched two gyro-radii above the guard cell and is considered to have left the system when the absolute value of the z-position reaches a distance of two gyro-radii above the launch point. Once all of the particles have been added together, the total moments are weighted such that the height integrated y-current density produces the proper change in the x-component of the magnetic field given by $f(z/l)$ (i.e. from -1 to +1). The total calculation may be greatly speeded up if we assume identical particle sources above and below the

current sheet, since we may then use the symmetry of the system to only launch particles from either above or below the current sheet and then reflect the moments across the $z=0$ plane. As a common first approximation, we assume that the electrons are sufficiently mobile so as to provide a neutralizing background. Non-symmetric sources may be modeled by launching separate distributions from either side of the current sheet.⁹ Once a new magnetic field has been calculated, it is compared with the initial field. If the fields are in agreement, we end the simulation, if not, we mix the old and new fields together and calculate a new input field. We typically use 95% old field and 5% new field to ensure convergence, but as a general rule, for smaller drift velocities a smaller percentage of the new field is required.

To convert a calculated equilibrium in code variables into physically meaningful quantities we begin by defining $L = \sigma R_E$, where R_E is the radius of the earth and σ is a constant to be determined. Assuming a proton plasma and using the definitions of the moments above, we find that $\sigma = 0.036 \sqrt{\left(\frac{\hat{n}_{top}}{n_0}\right)}$, where \hat{n}_{top} is the calculated density in the top grid cell and n_0 is the asymptotic particle density measured in cm^{-3} . Furthermore, the asymptotic magnetic field is given by $B_0 = 14.1 \sqrt{\frac{n_0 T_0}{\hat{n}_{top} \hat{T}}}$ where T_0 is the ion temperature measured in keV and \hat{T} is the ion temperature in code variables. The results for the case $n_0 = 0.35 cm^{-3}$, $T_0 = 5keV$, $E_{drift} = 0.1T_0$ and $B_z = 0.1B_0$ are shown in Figure 1 (a)-(c). The field looks very much like the Harris-magnetic field, but the density is almost constant throughout the reversal.

In figure 1(d) and (e) we show how the peak in the current density and the scale length of the field reversal vary as a function of the drift energy. Higher drift energies result in more peaked density profiles and thinner sheets. These properties may play an important role in the next phase of the project where we allow for more complex electron models. For example, if the electrons are taken as a Boltzmann distribution, the short scale length and peaked density may result in a significant electric field in the z -direction that will in turn modify the ion dynamics.

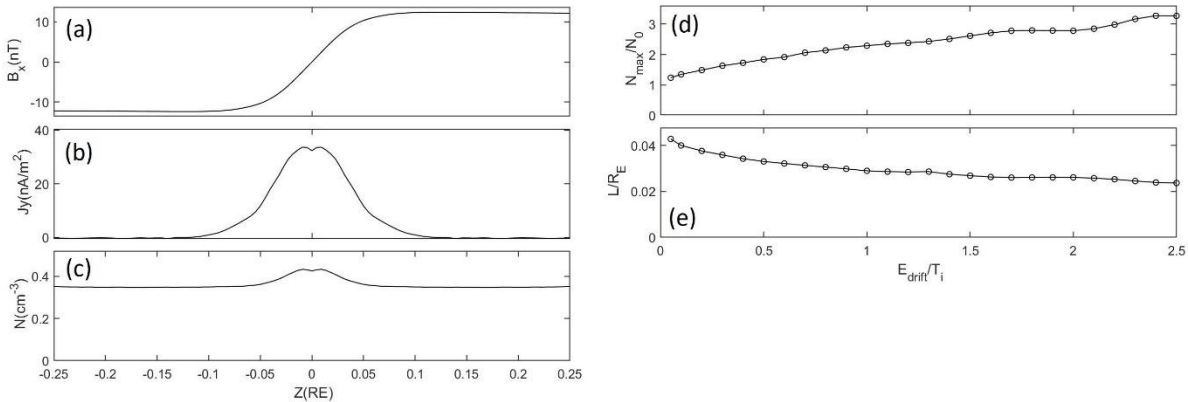


FIGURE 1. Example results from the code. (a)-(c) are the self-consistent magnetic field, current density and particle density for the case in which $n_0 = 0.35 cm^{-3}$, $T_0 = 5keV$, $E_{drift} = 0.1T_0$ and $B_z = 0.1B_0$. Frames (d) and (e) show the percentage increase in the peak density and the scale length of the field reversal as a function of the drift energy.

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