# A Brief Introduction to Plasma Accelerators

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**Abstract.** This work describes the physics of plasma waves and plasma accelerators. The superiority of plasma accelerators over conventional accelerators has generated renewed interest in these devices with the advent of ultra-fast laser technology. The ponderomotive force produced by ultra short laser pulses interacting with the plasma is considered and the resulting acceleration gradient is derived from first principles. Electron injection is described. Beam driven, including electron beam and proton beam, plasma waves are proposed as the future of high energy plasma accelerators.

**Keywords:** Plasma Physics, Plasma Waves, Ponderomotive Force, Ultrashort Lasers, Plasma Accelerators, Laser Plasma Accelerator, Electron injection
**PACS:** 52.75.-d; 52.35.-g

## INTRODUCTION

The primary limit of the current acceleration technology is the small accelerating gradient per meter which compounds the length and material cost of building more powerful accelerators.[1] The small accelerating gradient is due mainly to the maximum RF energy which can be tolerated by the accelerating structure before the structure degrades due to field emission.[1] Field emission happens when strong electromagnetic fields overcome the chemical potential of materials and rip off electrons from the surface, slowly degrading the material over time. However, plasma accelerators have the advantage that the electric field is immersed in a plasma, which significantly mitigates this effect. Plasma is the state of matter in which an ionized gas interacts with an equal number of free electrons in a confined space. The plasma accelerator's limits will be introduced later in this paper. The general idea of conventional particle accelerators is to accelerate charged particles by attracting and repelling them by a changing electromagnetic field. The current conventional accelerating gradient is about 100 MeV/m.[1] (The accelerating gradient is defined as the energy gained by the particle per unit length.) In a plasma accelerator the plasma acts as an energy transformer, where energy is transferred from the driver (the ultrashort pulse laser or high energy charged beam) to the accelerated particles. Using this regime plasma acceleration can achieve the acceleration gradient of 1 GeV/cm.

Laser-driven plasma accelerator was originally proposed three decades ago by Tajima and Dawson.[2] The basic design of a laser-driven plasma accelerator is applying an ultrashort laser pulse into plasma, creating a plasma wave. By injecting electrons onto the plasma wave, the electrons are accelerated.

In this paper, plasma acceleration, ponderomotive force, electron injection, beam-driven method, and some important milestones will be introduced.

## PLASMA ACCELERATION GRADIENT

To evaluate the plasma acceleration gradient, evaluating the maximum electric field that can be created in plasma is the first step. According to Maxwell's equations,

$$\int E \cdot ds = \int \frac{\rho}{\varepsilon_o} dV \qquad (1)$$

where $\rho$ is the charge density and $\varepsilon_o$ is the vacuum permittivity. In plasma, an electric field created by a charge that moved by distance x in a space of the charge density $n$ is

$$E = \frac{nex}{\varepsilon_o} \qquad (2)$$

Plasma frequency[3] is one of the most important parameters of the plasma, which is defined as the oscillation frequency of the plasma electrons, is given by

$$\omega_p^2 = \frac{ne^2}{\varepsilon_o m_e} \Rightarrow \frac{ne}{\varepsilon_o} = \frac{\omega_p^2 m_e}{e} \qquad (3)$$

Now replace $ne/\varepsilon_o$ in Eq. 2 by Eq. 3

$$E = \frac{\omega_p^2 m_e x}{e} \qquad (4)$$

The displacement of the electrons in the plasma cannot be larger than a plasma wavelength, because the plasma wavelength is the maximum displacement that plasma electrons can oscillate. The maximum field will happen when $x \sim \lambda_p$ ($\lambda_p$ is the plasma wavelength).

$$x \sim \lambda_p = \frac{c}{\omega_p} \qquad (5)$$

Thus, Eq. 4 becomes

$$E = \frac{\omega_p^2 m_e \left(\frac{c}{\omega_p}\right)}{e} \qquad (6)$$

Therefore,

$$eE_{max} = m_e c^2 \frac{\omega_p}{c} \quad (7)$$

By dimensional analysis, $eE$ can be understood as the accelerating gradient, both of which have the unit of energy per length. Recall the plasma frequency (3), when evaluated is approximately 9000 $n^{1/2}$ Hz (where n is the units of $cm^{-3}$) the maximum accelerating gradient then only depends on the plasma density.[3]

$$eE_{max} = 1 \frac{eV}{cm} \cdot n^{1/2} \quad (8)$$

One numerical understanding of this result is that 1 GeV/cm accelerating gradient will be achieved by plasma density $n = 10^{18}\ cm^{-3}$, corresponding to plasma wavelength $\lambda_p = 30\ \mu m$ (around 100 fs). The result of the required laser pulse width to create the plasma acceleration was published before pure femtosecond lasers were invented. Therefore, back in the early 1980s, laser pulse amplitude modulation (Figure 1) was utilized to create the shorter pulse widths associated with plasma wavelength.

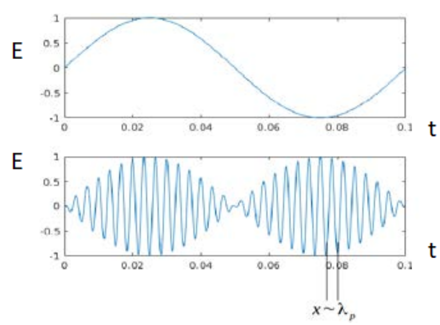


**FIGURE 1**. (a) Example of normalized laser pulse envelope without modulation. (b) Example of normalized amplitude modulated laser pulse and the associated where distance between the fast oscillation represents the plasma wavelength $\lambda_p$, such that the pulse is able to accelerate the plasma as shown in Figure 2.

## PONDEROMOTIVE FORCE

Light is able to exert a small amount of radiation pressure on an object proportional to the intensity of the incoming electromagnetic radiation. Electro-magnetic radiation that is used to heat plasma can also be coupled to particles in a non-linear fashion and the resulting force is called the ponderomotive force.[4]

Chen derives this force in the following fashion.[5] Consider an electron in an oscillating E and B field of an electromagnetic wave. The electron motion can be described by (9), where $m_e$ is the mass of a single electron.

$$m_e \frac{dv}{dt} = -e[E(r) + v \times B(r)] \quad (9)$$

The electric field will contain a spatial dependence that can be expanded to the second order about $r_0$ and time averaged. For a step by step description see Reference 5.

$$\mathrm{E} = E_s(r) \cos \omega t \quad (10)$$

Thus, substituting (10) into (9), expanding and time averaging the nonlinear force acting on the electrons by the pulse can be written as

$$f_{NL} = m_e \frac{dv_2}{dt}$$
$$= \frac{-e^2}{m_e \omega^2} \frac{1}{2} [(E_s \cdot \nabla) E_s + E_s \times (\nabla \times E_s)] \quad (11)$$

where $v_2$ is the second order velocity and $E_s$ is the amplitude of E in which contains the spatial dependence. Furthermore, (11) can be simplified by using the vector triple product as $E_s \times (\nabla \times E_s) = \nabla (E_s \cdot E_s) - E_s (E_s \cdot \nabla)$, canceling out the $E_s (E_s \cdot \nabla)$ terms.

We find that effective ponderomotive force on a single electron is:

$$f_{NL} = \frac{-1}{4} \frac{e^2}{(m_e \omega^2)} \nabla E_s^2 \quad (12)$$

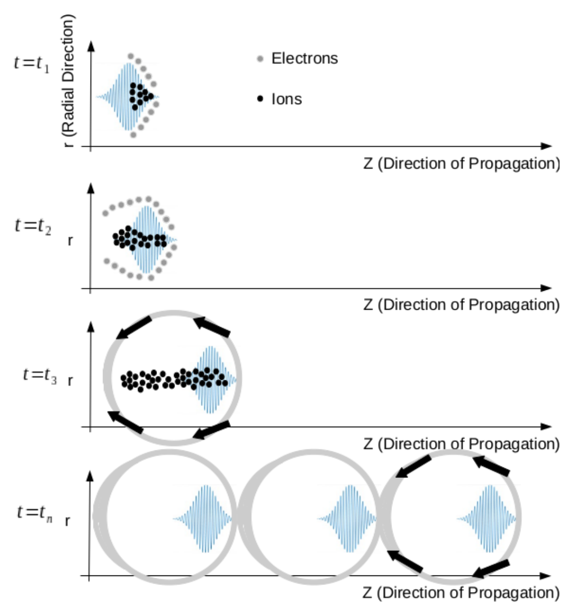


**FIGURE 2**. Plasma density altered by the massive EM field. At $t = t_1$, the electrons are repelled by the pulse and the ion at the center created a local positive region. At $t = t_2$, the electrons are attracted by the local positive region while the pulse is traveling. At $t = t_3$, the electron density form a bubble leaded by the pulse. At $t = t_n$, multiple bubbles are formed as a bubble train with multiple pulses.

By multiplying the force in (12) by the electron density, the ponderomotive force can be written in terms of plasma frequency. Plasma frequency $\omega_p$ is the rate at which electrons oscillate when they are displaced by a uniform background of ions generating a restoring force. The plasma frequency is therefore directly proportional to the density of the plasma and (3) can be rearranged and substituted into (12) to obtain the full ponderomotive force $F_{NL}$ as:

$$F_{NL} = \frac{-\omega_p^2}{\omega^2} \nabla \frac{\langle \epsilon_o E^2 \rangle}{2} \quad (13)$$

Electrons in a uniform field would oscillate purely in the direction of E but the magnetic field distorts their orbit. The Lorentz force acts in the direction of the wave number k which is in the direction of propagation, and if the amplitude of the electrical field varies, electrons will tend to bunch together or "pile up" in regions of smaller amplitude which becomes a local saddle point. The ponderomotive force acts directly upon electrons but the force is ultimately transmitted to ions due to charge separation field created by the space charge accumulated in the saddle points. The ions tend to flow toward the intensity minimum of the incoming radiation.

## ELECTRON INJECTION

As previously described, the plasma electrons are the medium of the plasma wave. These electrons only oscillate in the plasma instead of propagating. To accelerate these electrons, they need to be accurately injected into the plasma wave. There are two ways of injecting electrons, self-injection and external injection. Self-injection occurs through the wave breaking phenomenon. Some plasma electrons at the plasma wave front break out from the plasma wave during plasma wave propagation. These electrons are stuck in the region in between two bubbles and start traveling with the plasma wave (bubble train). Note that only the trapped electrons are accelerated; other plasma electrons are the wave medium which oscillate in the surrounding plasma. External injection is inserting electrons externally into the bubble train. This is experimentally challenging because the electron bunches must be placed in the right place and at the right time.[3]

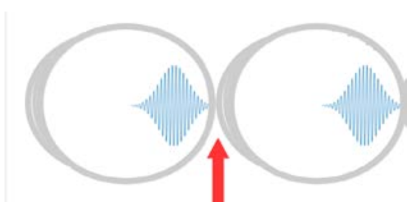

**FIGURE 3**. Position that electrons are injected with respect to the plasma wave is indicated by the red arrow.

## PARTICLE BEAM DRIVEN

As the laser pulse propagates through a plasma, several competing effects work against acceleration. Diffraction of light as it interacts with the plasma matter and dephasing between the laser pulse and the accelerated plasma are the two notable problems in laser-plasma acceleration. All of the above mentioned cause an overall depletion in laser energy. Particle beam driven solutions solve this problem since more energy can be carried by the particle beam. Others have proposed mitigating this effect by "staging" the accelerator into 10 GeV stages.[6] Experiments by the UCLA plasma accelerator group at SLAC (Stanford Linear Accelerator Center) showed that using an input electron beam of 42 GeV to drive the plasma acceleration was using to obtain energy doubling in a record breaking experiment.[7] Greater acceleration gradients are expected from the AWAKE (The Proton Driven Plasma Wakefield Acceleration Experiment) group in CERN.[8]

## CONCLUSION

Plasma acceleration utilizes the plasma as the energy transformer to transfer energy from the driver to the particle. Theoretically, it has a large acceleration gradient ~1 GeV/cm. The ponderomotive force creates the plasma wave. It is a non-linear radially decreasing distribution. One of the challenges in plasma acceleration is electron injection. Beam-driven plasma accelerators potentially will be the next generation accelerator due to the massive theoretical gradients possible, and its superiority over laser driven systems.

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